The Pitot Tube in MFD Flows

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Theme

THE Pitot tube is examined for possible application in MFD. Theoretical interpretations of the probe reading and a few illustrative experiments are reported for five different cases of undisturbed MFD flows in the full-length paper[1] and in further detail in an internal report.[2] Only the most important of these cases will be given here which is an extension of an existing theory[3-5] for an incompressible flow to its subsonic compressible equivalent. This kind of flow occurs typically in MHD channels for power generation. A universally applicable, purely experimental method is suggested which circumvents certain difficulties involved in the application of such theories.

Contents

Theoretically, the Pitot pressure, which is the pressure at the forward stagnation point of a slender Pitot tube aligned with the flow, may be calculated by integrating the momentum equation along the stagnation streamline. The Lorentz force in the momentum equation and the Ohmic heating in the energy equation will cause electromagnetic probe errors in MFD flows. Similar frictional probe errors have been studied intensively in the literature, and are discussed in the full paper.[1,2]

A probe theory valid for arbitrary MFD flows is not yet available and only simple MFD flows can be studied. The one chosen here is represented by

$$\boldsymbol{B}_\infty \perp \boldsymbol{v}_\infty, \quad \boldsymbol{j}_\infty = 0, \quad \boldsymbol{E}_\infty = -\boldsymbol{v}_\infty \times \boldsymbol{B}_\infty$$

such that in the x, y, z coordinate system of Fig. 1, one has,

$$\boldsymbol{v}_\infty = v_\infty \boldsymbol{e}_x, \quad \boldsymbol{B}_\infty = B_\infty \boldsymbol{e}_y, \quad \boldsymbol{E}_\infty = -v_\infty B_\infty \boldsymbol{e}_z, \quad \boldsymbol{j}_\infty = 0 \qquad (1)$$

where subscript ∞ stands for the flow undisturbed by the probe. No index will denote the flowfield with the probe inserted. All quantities in Eq. (1) are assumed constant in space and time. For simplicity, it is assumed that the electrical conductivity of the flow $\sigma = \text{const}$, and that the fluid is an ideal gas. Finally, the magnetic Reynolds number is assumed to be small.

Under these conditions, the following MFD equations may be derived[1,2] for the flow with the probe.

$$\mathrm{d}p = -(\gamma M_\infty^2 p)^{1/n} v \mathrm{d}v + N(\boldsymbol{j} \times \boldsymbol{B}_\infty)_x \mathrm{d}x \quad \text{(momentum)} \qquad (2)$$

$$n = \gamma \left[1 - (\gamma - 1) \frac{N \int_{-\infty}^{x_s} \frac{1}{\rho v} j^2 \mathrm{d}x}{\frac{1}{2} + \int_{-\infty}^{x_s} \frac{1}{\rho} (\boldsymbol{j} \times \boldsymbol{B}_\infty)_x \mathrm{d}x} \right]^{-1} \qquad (3)$$

$$\boldsymbol{j} = \boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}_\infty \quad \text{(Ohm's law)} \qquad (4)$$

$$\boldsymbol{E} = \nabla \phi \qquad (5)$$

These equations were nondimensionalized by referring x to d, v to v_∞, ρ to ρ_∞, p to $\rho_\infty v_\infty^2$, $\boldsymbol{j}$ to $\sigma v_\infty B_\infty$, $\boldsymbol{B}$ to B_∞, and $\boldsymbol{E}$ to $v_\infty B_\infty$. The symbol $\boldsymbol{B}_\infty$ is retained for identification although in dimensionless form $\boldsymbol{B} = \boldsymbol{B}_\infty = \boldsymbol{e}_y = \text{const}$ according to the assumptions. $N \equiv (\sigma B_\infty^2 d)/(\rho_\infty v_\infty)$ is the interaction parameter and γ is the isentropic exponent. Equations (2) and (3) are given as evaluated along the stagnation streamline in front of the probe and n is a mean polytropic exponent defined for the thermodynamic change of state between the undisturbed flow far from the probe ($x = -\infty$, $v = 1$) and the stagnation point ($x = x_s$, $v = 0$). It permits the formal integration of Eq. (2). Equation (3) then replaces the energy equation. Ohm's law, Eq. (4), is taken in its classical form, but the influence of the Hall effect is discussed in the full paper.[1,2] According to Eq. (3) the Lorentz force cannot by itself cause any departure from isentropy ($n = \gamma$) in the stagnation process but only through the heat dissipation, j^2/σ.

Since for most applications of Pitot tubes in MFD flows one can expect $N < 1$ to be true, all unknown quantities ($\boldsymbol{v}, \rho, \boldsymbol{E}, \phi, \boldsymbol{j}$) may be developed in a power series of N

$$A = A_0 + A_1 N + A_2 N^2 + \ldots \qquad (6)$$

Using Eq. (6) for $\boldsymbol{v}, \rho$, and $\boldsymbol{j}$ in Eqs. (2) and 3), up to first order in N one finds

$$\frac{\mathrm{d}p}{\mathrm{d}x} = -\frac{\mathrm{d}(v_0^2/2)}{\mathrm{d}x} (\gamma M_\infty^2 p) \exp\left(\frac{1}{\gamma} \left[1 - \left\{ 2(\gamma - 1) \int_{-\infty}^{x_s} \frac{1}{\rho_0 v_0} j_0^2 \mathrm{d}x \right\} N \right] \right) + N(\boldsymbol{j}_0 \times \boldsymbol{B}_\infty)_x \qquad (7)$$

v_0 and ρ_0 are the velocity and the density fields, respectively, with the probe in the flow, but without the electromagnetic effects, and should be known from ordinary gasdynamics (OGD). In order to calculate $\boldsymbol{j}_0$ from $\boldsymbol{v}_0$, the quantities $\boldsymbol{j}$, $\boldsymbol{E}$, $\boldsymbol{v}$, and ϕ are expanded according to Eq. (6), and inserted in Eqs. (4) and (5), and in the divergence of Eq. (4), $\nabla^2 \phi = \boldsymbol{B}_\infty \cdot \operatorname{curl} \boldsymbol{v}$. Equating coefficients of equal powers of N in the resulting equations, one obtains for the coefficients of N°

$$\nabla^2 \phi_0 = 0, \quad \boldsymbol{E}_0 = \nabla \phi_0, \quad \boldsymbol{j}_0 = \boldsymbol{E}_0 + \boldsymbol{v}_0 \times \boldsymbol{B}_\infty \qquad (8)$$

In a similar manner,[1,2] the boundary conditions necessary for the solution of Laplace's equation $\nabla^2 \phi_0 = 0$ are found to be

$$\phi_0 = -z \quad \text{at infinity}$$

$$\frac{\partial \phi_0}{\partial m} + (\boldsymbol{v}_0 \times \boldsymbol{B}_\infty)_m = 0 \quad \text{on the probe surface} \qquad (9)$$

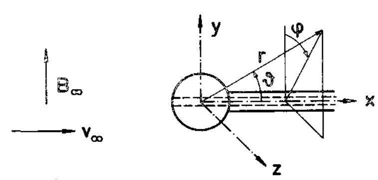

Fig. 1 Pitot probe and coordinate systems.

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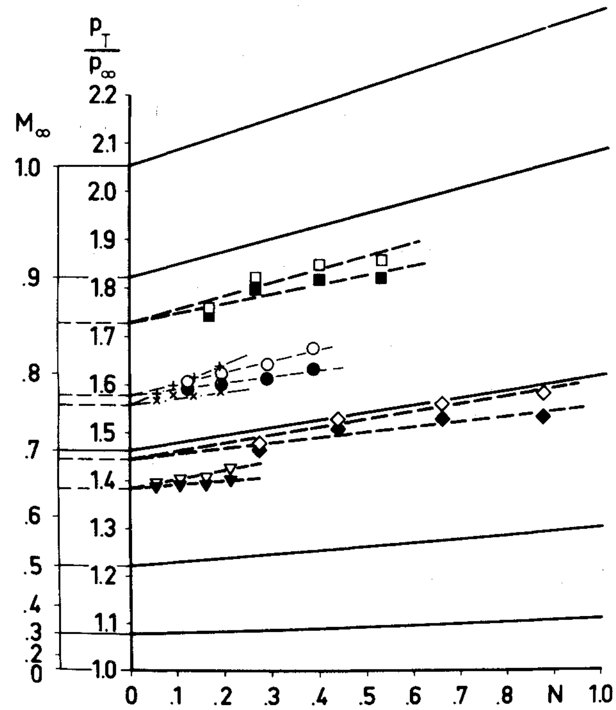

Fig. 2 Pitot pressure variation with Mach number, M_∞, and interaction parameter, N.

This means that at infinity the flow is not disturbed by the probe and the probe itself is chosen to be electrically nonconductive (m denotes the direction perpendicular to the surface of the probe).

Because the compressible subsonic flow around a sphere, and thus v_0 and ρ_0, are known from OGD,[6] a probe with a spherical head on a thin stem with negligible influence on the flow was chosen as an example. Using the spherical coordinate system r, φ, ϑ as shown in Fig. 1, the solution of the boundary value problem for ϕ_0 is

$$\phi_0 = -z + \frac{1}{4} r^{-2} \sin\vartheta \sin\varphi + M_\infty^2 \left[-\frac{7}{80} r^{-2} \sin\vartheta \sin\varphi + \frac{8}{28160} r^{-4} (15 \sin 3\vartheta + 3 \sin\vartheta) \sin\varphi \right] \quad (10)$$

The dimensionless r refers to the radius of the sphere, R. The j_0-field is calculated from Eqs. (8) and (10).

Equation (7) is now transformed to the r, φ, ϑ system and the v_0, ρ_0, and j_0 distributions along the stagnation streamline system ($\varphi = 0$, $\vartheta = \pi$) inserted. The resulting equation can then be integrated numerically along the stagnation streamline between $r = \infty$ ($p = p_\infty$) and $r = 1$ ($p = p_T$). The result, shown as solid lines in Fig. 2, represents the desired theoretical Pitot pressure to a first approximation, as a function of the parameters M_∞ and N. If $B_\infty = 0$ or $\sigma = 0$, i.e., $N = 0$, then the electromagnetic effects on the Pitot pressure, p_T, dissappear and the probe measures the error-free (gasdynamic) Pitot pressure, $p_{T\,id}$. The probe error may then be defined as $(p_T/p_\infty) - (p_{T\,id}/p_\infty)$. The results of some experiments also are exhibited in Fig. 1 and fall between the theoretical curves, thus supporting the theory.

The previous theory is restricted to $0 < M_\infty < 1$[6] and the first approximation is good for $0 < N < 0.3$ only. Within these limits, it permits the determination of the electromagnetic error, once M_∞ and N are known. M_∞ and v_∞ in N, however, are the quantities actually sought in a Pitot measurement. Iterative and other methods[2] to find v_∞ with the aid of this theory in the incompressible case fail for compressible flows. The following purely experimental method may instead be applied.

If the diameter of the Pitot tube is made arbitrarily small, then it may be shown from the momentum and the energy equations that the electromagnetic effects dissappear, and the frictional and the heat transfer effects grow infinitely large while the Euler terms remain unchanged. Measuring with a set of Pitot tubes of the same geometric shapes, but with decreasing d, one may therefore extrapolate the curve $p_T(d)$ to $d \to 0$ to obtain the Pitot pressure free of electromagnetic errors. The only requirement is that frictional and heat-transfer terms are negligible for the probes which means that the probes should be "large enough." Complications with the limits of validity of the continuum-mechanical equations as $d \to 0$ are then also excluded. Note that a set of probes is needed in MFD, contrary to one probe only in OGD, to measure the true Pitot pressure.

The procedure may be applied for any configuration and distribution of $\boldsymbol{j}_\infty$, $\boldsymbol{E}_\infty$, $\boldsymbol{B}_\infty$, $\boldsymbol{v}_\infty$ in the undisturbed flow. It is suitable for all Re_m, M_∞, and N. Since the extrapolation is made over d rather than N, N must not be known beforehand. If the electrical conductivity of the probes were identical for all probes, the procedure would appear to be independent of the probe conductivity, also. Experience with mercury flows[3] and plasma flows,[1,2] however, indicates that uncontrollable and time-dependent contact resistances on the probe surface render the method inapplicable to conductive probes. The use of insulated probes is therefore recommended.

The universal method as outlined above was applied for the special MFD flow of Eq. (1). To permit a comparison with the theory, the extrapolation was made over N to $N = 0$ rather than over d to $d = 0$ ($N \sim d$), in Fig. 2. The error-free Pitot pressure may be read at the intersection of the extrapolated experimental curve with the (p_T/p_∞)-axis at $N = 0$. The corresponding Mach number M_∞ is also indicated for $\gamma = 1.67$ (argon was used in the experiments).

References

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